Math 142 **Test #2** **Fall 2011**

You must show all your work to receive full credit.

1. (8 pts) Recall that $\sinh x = \frac{1}{2}(e^x - e^{-x})$ and that $\cosh x = \frac{1}{2}(e^x + e^{-x})$. Evaluate each of the following expressions.
   (a) (2 pts) $\sinh 1$
   (b) (3 pts) $\tanh(\ln 2)$
   (c) (3 pts) $\frac{d}{dx}(\sinh x)$

2. (10 pts) Find the following indefinite integrals.
   (a) (5 pts) $\displaystyle\int \frac{1}{25 - x^2}\, dx$
   (b) (5 pts) $\displaystyle\int \frac{2}{3x\sqrt{9 + x^2}}\, dx$

3. (14 pts) Given the differential equation
$$\frac{dy}{dx} = 2x(y + 2),$$
   (a) (10 pts) Find the general solution.
   (b) (4 pts) Find the particular solution that satisfies the initial condition $y(0) = 4$.

4. (15 pts) The region bounded by $x = 2$, $x = 4$, $y = 0$, and $y = x$ is revolved about the x-axis, generating a solid of revolution. Use integration to find the volume of the solid.

5. (12 pts) The region in the 1st quadrant bounded by $y = 9 - x^2$ is revolved about the y-axis, generating a solid of revolution. Use the shell method to find the volume of the solid.

6. (8 pts) Set up the integral that represents the arclength of the curve $y = x^3$ from $x = 0$ to $x = 1$. Do not evaluate the integral.

7. (15 pts) An empty cylindrical tank is 10 feet long and 4 feet in diameter. It is lying on its side on the ground. How much work is done in filling the tank half full of water by pumping water into it from ground level? The density of water is $\rho = 62.4$ lb/ft^3. Hint: Break the integral up into two integrals, one of which you can evaluate by using a simple geometric formula.

8. (18 pts) Find the center of mass of the lamina of uniform density ρ bounded by the graphs of $y = x^2$ and $y = 9$. Obviously, $M_y = 0$. Do the rest in three parts.
   (a) (10 pts) Find M_x, the first moment about the x-axis.
   (b) (5 pts) Find m, the mass.
   (c) (3 pts) Find $(\overline{x}, \overline{y})$, the center of mass.